Eliciting Choice Correspondences

A General Methodology and an Experimental Implementation

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Introduction

Utility, Preferences and Choices

- Economists are interested in the understanding of individual choice
- A decision maker is often modeled with a *preference* (or a utility)
- Preferences can be constructed from observed choices, using the revealed preference approach of Samuelson (1938)
- The condition for constructing a "rational" preference from observed choices is the Weak Axiom of Revealed Preferences (WARP) (Arrow 1959)
- WARP often fails, but there are **regularities** in these failures

WARP

If *tea* is strictly revealed preferred to *coffee*, then *coffee* cannot be revealed preferred to *tea*.

Question

Research question

Is it possible to (and how can one) go from observed choices to preferences, when WARP fails?

Issue Here

- In theory, we need a **choice correspondence** (c^c) to do so
- In practice, experiments elicit a **choice function** (c^f), which limits the ability to explore failures of WARP

Main challenge

How can one identify the choice correspondence of a decision maker?

A Motivating Example

Observed choices of a decision maker with preferences coffee $\sim$ tea $\sim$ hot chocolate in all subsets of $X = \{C, T, H\}$

- $c(\{C, T\}) = \{C\}$; Preferences : coffee $\succeq$ tea
- $c(\{C, H\}) = \{H\}$; Preferences : hot chocolate $\succeq$ coffee
- $c(\{T, H\}) = \{T\}$; Preferences : tea $\succeq$ hot chocolate
- $c(\{C, T, H\}) = \{T\}$; Preferences : tea $\succeq$ coffee and $\succeq$ hot chocolate

Revealed Preferences

coffee $\sim$ tea $\sim$ hot chocolate and hot chocolate $\succ$ coffee, which is "irrational", as it does not verify WARP.

Contributions

Four main contributions:

1. **Pay-for-certainty**, a direct and general methodology for identifying choice correspondences
2. Explore which models rationalize observed choices when WARP fails
3. Show that indifference matters in practice, which matters for *individual* welfare analysis
4. Show that choice correspondence help aggregate choices and preferences, which is useful for *collective* welfare analysis

Main Results

- **3%** of subjects behave always choose singletons, and 60% of observed choices are *not* singletons
- The choice correspondence is identified for **26%** of subjects, bounded for 72% of subjects
- WARP is verified by **47%** of observed choices
- **93%** of observed choices can be rationalized by a preference
- Indifference is widespread, it represents between **36%** and **50%** of comparisons between alternatives

Related Literature

- Choice correspondences in theory: Sen (1971), **Eliaz and Ok (2006),** Nosratabadi (2017), **Aleskerov, Bouyssou, and Monjardet (2007), Frick (2016)** and many others
- Choice correspondences in practice: indirect elicitation in Danan and Ziegelmeyer (2006), **Costa-Gomes et al. (2016),** Agranov and Ortoleva (2017), Qiu and Ong (2017)
- Revealed preference axioms in experiments: Choi et al. (2007), Choi et al. (2014), Bouacida and Martin (2017) and many others, all of them with *choice functions*
- Aggregation of incomplete preferences: Danan, Gajdos, and Tallon (2013), Danan et al. (2016)

Methodology

General Setting and Notations

General problem : We have a finite set of alternatives X and a decision maker with *transitive strict*, but NOT necessarily complete preferences $\succeq$ on X.

How can we obtain the preference from observed choices?

Hereafter, S is a non empty, non singleton element of 2^X ($S \subseteq X$)

Set of *maximal* alternatives:

$$M(S) = \{x \in S | \nexists y \in S, y \succ x\}$$

Revealed Preferences

Identifying a Choice Correspondence?

Problem With indifference, simply allowing multiple choices does not work!

Example Take X = {coffee, tea}, with coffee ∼ tea, M(X) = {coffee, tea}. What is $c(X)$?

- {coffee}?
- {tea}?
- {coffee, tea}?

Result

$$\forall S \in 2^X, c(S) \subseteq M(S)$$

Costly Certainty

Example Take X = {coffee, tea}, with coffee $\sim$ tea, M(X) = {coffee, tea}. The DM earns ε (small) by selecting an alternative. What is $c_\varepsilon(X)$?

- ~~{coffee}~~
- ~~{tea}~~
- **{coffee, tea}**

$c_\varepsilon(X)$ is unique and $c_\varepsilon(X) = \{\text{coffee, tea}\} = M(X)$!

Generalization: Pay-For-Certainty

A new methodology to elicit a choice correspondence, *pay-for-certainty*:

- For any set of alternatives S
- Selecting one alternative adds an additional payment of ε (small)
- The alternative given to the decision maker is selected using a selection mechanism, here, a uniform random draw

Question Under which condition the pay-for-certainty methodology identifies the choice correspondence c?

Extended Setting

- $X \times \mathbb{R}^+$ is the set of alternatives associated with a gain: $(x, r) \in X \times \mathbb{R}^+$
- $\succeq_2$ is the preference of the decision maker on $X \times \mathbb{R}^+$
- c_ε the choice correspondence when the gain of adding an alternative is $\frac{1}{|S|}\varepsilon$

Link Between $\succeq$ and $\succeq_2$

Identity Observed preferences with no gain and unobserved preferences are identical

$$x \succeq y \Leftrightarrow (x,0) \succeq_2 (y,0)$$

Monotonicity For a given alternative x in X, the highest gain is strictly preferred to the lowest one

$$\forall x \in X, \forall r, r' \in \mathbb{R}^+, r > r' \Leftrightarrow (x,r) \succ_2 (x,r')$$

Transitivity of strict observed preferences $\succ_2$ is transitive

Why $M_\varepsilon = c_\varepsilon$?

Strict r-domination An alternative x strictly r-dominates an alternatives y iff $(x, 0) \succ (y, r)$

$$M_\varepsilon(S) = \left\{ x \in S : \nexists y \in S, (y, 0) \succ_2 \left(x, \frac{1}{|S|}\varepsilon \right) \right\}$$

Constrained Maximization of the gain of the chosen set In all sets of non $\frac{1}{|S|}\varepsilon$-dominated alternatives, the decision maker chooses the more rewarding one, i.e: the largest one

Under constrained maximization of the gain of the chosen set:

$$c_\varepsilon(S) = M_\varepsilon(S)$$

Increasing Chosen Set

When the decision maker has *transitive*, *monotone* (wrt to gain) preferences and *maximizes the gain*, she follows **increasing chosen set**:

$$\forall S \in 2^X \backslash \emptyset, \forall \varepsilon, \varepsilon' \in \mathbb{R}^+, \varepsilon < \varepsilon' \Rightarrow c_\varepsilon(S) \subseteq c_{\varepsilon'}(S)$$

Identification of the Choice Correspondence

The choice correspondence of the decision maker is **partially identified** when ICS is verified. We have:

$$\forall \varepsilon > 0, c_0(S) \subseteq M(S) = c(S) \subseteq c_\varepsilon(S)$$

The choice correspondence of the decision maker is **fully identified** when:

$$\forall S \in 2^X \setminus \emptyset, \exists \varepsilon > 0, c_\varepsilon(S) = c_0(S) = c(S)$$

Experiment

Outline of the Experiment

1. Instructions, including a description of the tasks: a sentence, a video or a video + training, depending on the treatment
2. Choosing a task: 33 choices in total
3. Risk aversion elicitation, using Dohmen et al. (2011)
4. Non-incentivized questionnaire
5. Performing **one** task

Choosing a Task

- Alternatives: **4** paid tasks: addition, spell-check, copy and memory
- Between subjects treatments: **3** different treatments, on the information provided about the tasks (sentence, video and video + training)
- Within subjects treatments, **3** different gains: 0 cent (no gain), 1 cent (low), 12 cents (high), some subjects faced only no and low gains
- Power-set elicitation: subjects chose in all possible subsets of alternatives: **11** choices
- In total, subjects made 3 times 11 choices: 33 choices

Screenshots

Data

223 subjects in total:

- 102 subjects in the video treatment
- 33 subjects in the sentence treatment
- 37 subjects in the video + training treatment
- 51 subjects choose according to a choice function. The instructions were provided as in the video treatment
- Average gain in the task: 3.26€ Histogram

Proportion of Singletons, by Gain

Table 1: Proportion of singleton chosen, by gain and choice sets sizes Differences between no gains and positive gains are significant. Differences between low and high gains are not.

Choice set size	No	Low	High
2	**65.6%**	**47.3%**	**46.5%**
3	39.9%	20.5%	18.6%
4	27.0%	11.1%	11.6%

Conclusion Size of chosen set is not 1 in general!

Proportion of Singletons, by Information

Table 2: Proportion of singleton chosen, by information provided

Choice set size	Sentence	Video	Training
2	**40.1%**	**54.8%**	**60.8%**
3	15.4%	28.9%	29.5%
4	8.1%	18.8%	18.0%

Conclusion The more precise the information is, the more "precise" the belief is!

Identification

Table 3: Proportion of subjects following increasing chosen sets between no and positive gains

Treatment	Sentence	Video	Training	All
Partial	88%	68%	73%	**72%**
Full	27%	24%	32%	**26%**

By cost pairs

Distance from Full Identification

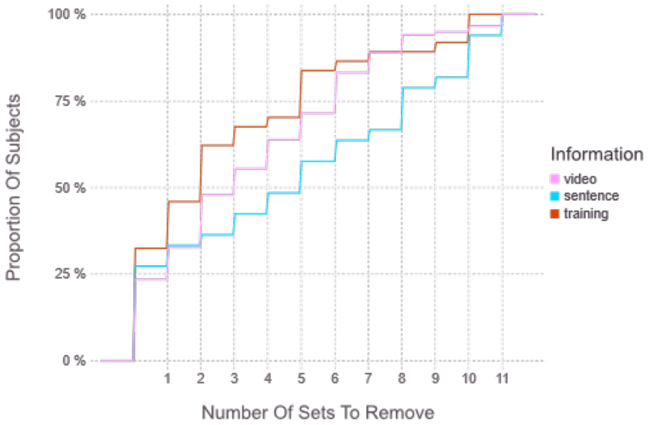

Figure 1: Number of sets to remove to fully identify the choice correspondence. The CDF are not significantly different from each others, but the means are.

Distance from Partial Identification

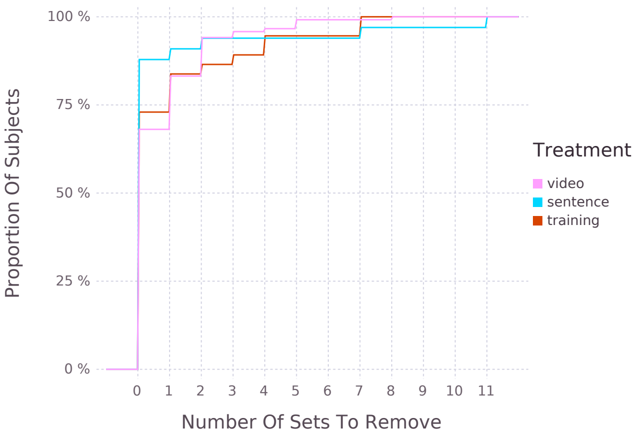

Figure 2: Number of sets to remove to partially identify the choice correspondence

Revealed Preferences and Choice Correspondences

WARP

WARP If x is chosen when y is available in one set, then if y is chosen in any other set, x must be too

Rationalizability

WARP is equivalent to having a complete, transitive and reflexive preference (to be rationalized by a classical preference)

- x is *revealed preferred* to y if x is chosen when y is available
- x is *strictly revealed preferred* to y if x is chosen and y is not chosen
- x and y are *indifferent* if both are chosen together

Empirically

Table 4: WARP verification

	Function	ε-correspondence	Partial	Identified
WARP	57%	46%	58%	98%**

Table 5: WARP on ε-correspondences, by information provided

Treatment	Sentence	Video	Training
WARP	62%***	42%	45%
Gain	2.66**	3.23	3.88**

WARP and Increasing Chosen Sets

Table 6: WARP and increasing chosen set

		WARP		
		all gains	some gains	never
	all gains	**20%**	19%	2%
ICS	some gains	1%	**36%**	7%
	never	0%	5%	**18%**

WARP Violations

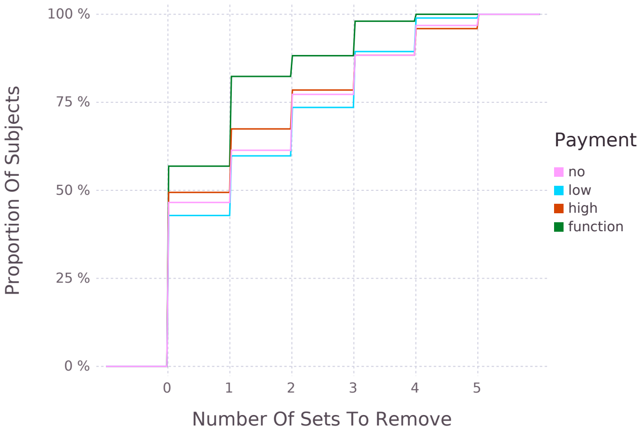

Figure 3: Number of sets to remove to verify WARP.

Relaxations of Transitivity or Completeness

With choice correspondences, investigations of relaxations of weak orders are possible:

- Models of intransitive indifference, aka semi-orders (Luce (1956)) and interval orders (Fishburn (1970))
- Models of incomplete preferences, aka partial orders (Aumann (1962), Eliaz and Ok (2006))

Revealed Incomplete Preferences

WARNI (Eliaz and Ok 2006)

$$\forall S \in 2^X, y \in S,$$
$$\text{if } \forall x \in c(S), \exists T \in 2^X, y \in c(T), x \in T \Rightarrow y \in c(S)$$

For a given alternative y in S, if for all the chosen alternatives in S, there exists a set T where x is in T and y is chosen in T, then y must be chosen in S. This property should be true for all S and y

WARNI is equivalent to rationalizability by a *partial order* (transitive and reflexive preference).

Revealed Intransitive Indifference (Aleskerov, Bouyssou, and Monjardet (2007)) I

Functional Asymmetry

$$\forall S, S' \in 2^X,$$
$$c(S) \cap (S' \backslash c(S')) \neq \emptyset \Rightarrow c(S') \cap (S \backslash c(S)) = \emptyset$$

If some chosen alternatives in S are not chosen in S', it must be that all chosen alternatives in S' that are in S are chosen.

Semi-orders rationalizability: partial order rationalizability and Functional Asymmetry

Revealed Intransitive Indifference (Aleskerov, Bouyssou, and Monjardet (2007)) II

Jamison-Lau-Fishburn

$$\forall S, S', S'' \in 2^X, S \subseteq (S' \backslash c(S')), \\ c(S') \cap S'' \neq \emptyset \Rightarrow c(S'') \cap (S \backslash c(S)) = \emptyset$$

If S is made of unchosen alternatives in S' and S'' counts some chosen alternatives in S', then chosen alternatives in S'' must be chosen in S.

Interval orders rationalizability: semi-order rationalizability and Jamison-Lau-Fishburn

Weakenings of Classical Preferences in Practice

Table 7: Rationalizability by different weakenings of WARP

Choice	ε-correspondence	Identified
Weak Order (WARP)	46%	98%***
Semi Order	49%	98%***
Interval Order	49%	98%***
Partial Order (WARNI)	49%	98%***

Conclusion Direct weakenings of the classical paradigm do not rationalize significantly more than WARP.

Threshold Rationalizability

General Threshold Representation A choice correspondence c on X admits a threshold representation if there exist functions $u : X \to R$ and $t : X \times X \times 2^X \to \mathbb{R}^+$ such that for every S,

$$c(S) = \left\{ x \in S \,\middle|\, \max_{y \in A} u(y) - u(x) \leq t(x, y, S) \right\}$$

u is the fully rational benchmark and t the departure threshold of the representation. t is a function. Here it will depend on any combination of x, y and S.

Monotone Threshold

Axiom: *Occasional Optimality* Frick (2016) For all S, there exists $x \in c(S)$ such that for any S' containing x:

1. If $c(S') \cap S \neq \emptyset$, then $x \in c(S')$.
2. If y is in S, then $c(S') \subseteq c(S' \cup \{y\})$.

A choice correspondence verifying occasional optimality is rationalizable by a monotone threshold representation

Menu-Dependent Choice

Axiom *Functional Asymmetry* Aleskerov, Bouyssou, and Monjardet (2007)

A choice correspondence c verifies *functional asymmetry* iff

$$\text{for all } S, S' \in 2^X, c(S) \cap (S' \backslash c(S')) \neq \emptyset \Rightarrow c(S') \cap (S \backslash c(S)) = \emptyset$$

If some chosen alternatives in S are not chosen in S', it must be that all chosen alternatives in S' that are in S are chosen.

A choice correspondence verifying functional asymmetry is rationalizable by a menu-dependent threshold representation

FA

When a choice correspondences verifies FA, its strict revealed preferences are acyclic

Context-Dependent Choice

Axiom *Fixed Point* Aleskerov, Bouyssou, and Monjardet (2007) A choice correspondence satisfies *fixed point* if for any $S \in 2^X$, there exist an alternative x in S such that x in S' implies that x in $c(S')$ for any $S' \subseteq S$.

A choice correspondence verifying fixed point is rationalizable by a context-dependent threshold representation

FP

When a choice correspondences verifies FP, it is possible to build complete, reflexive and transitive preferences

Empirically

Table 8: Varying threshold rationalizabilities. Significance levels are with semi-orders.

	ε-correspondence	Partially	Fully
classical preferences	46%	59%	98%
semi-order	49%	59%	98%
monotone threshold	60%***	69%	98%
menu-dependent threshold	88%***	89%***	100%
context dependent threshold	93%***	91%***	100%

Conclusion Introducing menu-dependence allow the reconstruction of a utility for most observed choices

Indifference

Indifference on Choice Functions

For a choice function, in principle, we should use the **weak** assumption of revealed preferences.

When both x and y are available in a given set:

- x is chosen means that x is weakly better than y (xRy)
- x is strictly better than y (xPy) if and only if we never have y weakly better than x
- x and y are indifferent (xIy) when xRy and yRx

Indifference

Table 9: Indifference and strict preferences, for choice functions, correspondences and ε-correspondence. Significance levels are with choice correspondences which verify WARP.

Axiom	Choice	Indifference	Strict Preference
WARP	Function	0.29***	5.71***
WARP	Correspondence	2.96	3.04
WARP	ε-correspondence	3.00	3.00
FP	ε-correspondence	2.15***	3.85***

Conclusion: Significant indifference is recovered with choice correspondences

Indifference by Information

Table 10: Average number of indifference relations with ε-correspondences and choice correspondences, by information provided. Significance levels are with choice correspondences which verify WARP.

Axiom	Choice	Sentence	Video	Training
WARP	Function		0.29***	
WARP	Correspondence	4.33	2.61	2.73
WARP	ε-correspondence	3.98	2.81	2.36
FP	ε-correspondence	3.53***	1.36***	1.07**

Indifference Relations

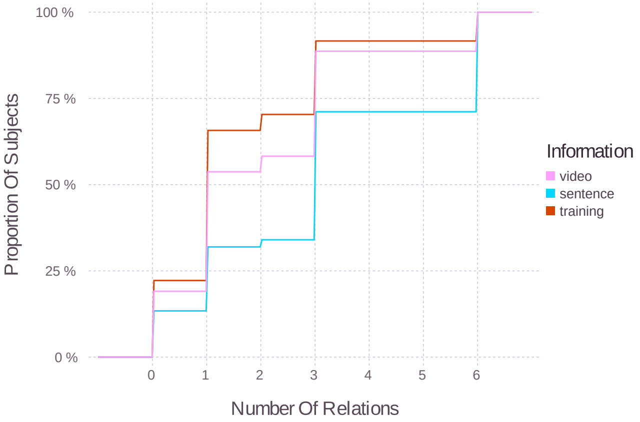

Figure 4: CDF of the number of indifference relations, by information provided. The p-value of the Mann-Whitney test of the difference between the training and the video treatment is 0.055. For the sentence and video treatment, it is 0.0001

Aggregation

A Non Preference Based Approach

Condorcet winner An alternative x is a *Condorcet winner* for a correspondence c if it is chosen in all binary choices with other alternatives from X.

$$\text{for all} y \in X, y \neq x, x \in c(\{x, y\})$$

Then approval voting procedure, where all Condorcet winners are approved.

Maximal Alternatives

Consider a preference, take all maximal alternatives $M(S)$

Then approval voting: all maximal alternatives are approved

Condorcet winners

Table 11: Which alternative is the Condorcet winner? Average of each alternative being a winner, depending on the choice procedure considered. Significance levels are with respect to choice correspondences.

	Addition	Spellcheck	Memory	Copy
Choice function	29%***	24%***	22%***	24%***
Choice correspondence	73%	73%	65%	69%
ε-correspondence	65%	56%	58%**	68%

Maximal Alternatives

Table 12: Which alternative is maximal? Average of each alternative being maximal, depending on the choice procedure considered. Significance levels are given with respect to choice correspondences. ε-correspondences when WARP or FP is verified are significantly different from each others.

Axiom	Choice	Addition	Spellcheck	Memory	Copy
WARP	Function	24%***	24%***	21%***	31%***
WARP	Correspondence	73%	73%	65%	69%
WARP	ε-correspondence	70%	68%	65%	74%
FP	ε-correspondence	48%***	40%***	44%***	52%**

Aggregation by Information

Table 13: Proportion of ε-correspondences for which a given alternative is a Condorcet winner, depending on the information provided. Significance levels are given for a task, between the treatments.

Treatment	Addition	Spellcheck	Memory	Copy
Sentence	72%	62%*	**85%*** **	75%*
Video	64%	51%	**58%**	65%
Training	59%	65%***	**36%*** **	72%

Discussion and Conclusion

Conclusion

- Introduce and implement a methodology for identifying choice correspondences: *pay-for-certainty*
- Subjects choose more than one alternative when allowed to do so: only 3% ε-correspondences are choice functions
- Indifference is directly uncovered and is significant
- Incomplete preferences do not show up in my experiment, contrary to elsewhere in the literature
- Using context-dependent models allow me to get a preference in most cases
- It is easier to aggregate preferences with choice correspondences

Thank you

Thank you for your attention! Questions?

Open(ed) questions?

- Influence of the kind of alternatives on incompleteness
- Robustness with respect to the selection mechanism
- Robustness with respect to the incentive to choose more than one alternative

Biases and Robustness

Some effects:

- Small learning effect
- Allowing for one or two mistakes significantly improve consistency
- Looking at the training treatment: small correlation with the choice

Controls:

- No effect of the order of alternatives
- No effect of risk aversion
- Expected gains and variance of the gains are not significantly different between the different tasks

Appendix

Choice Screen

Figure 5: Choice screen

Confirmation Screen

Introduction Experiment

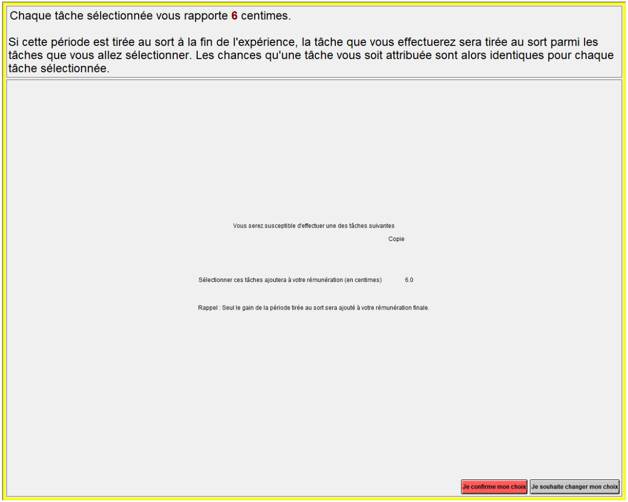

Figure 6: Confirmation screen

Gains in the Tasks

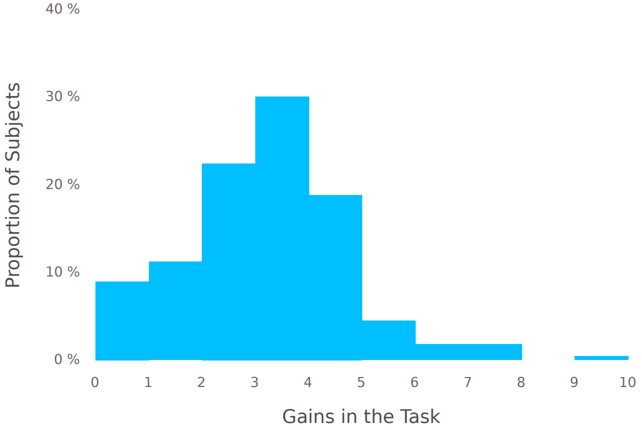

Figure 7: Histogram of the gains in the tasks

Gains by Task

Table 14: Average gains depending on the task, in €

	Addition	Spell-check	Memory	Copy
Mean (std)	3.30 (1.84)	3.05 (1.68)	2.80 (1.21)	3.34 (1.27)

None of the gains are significantly different, except copy and memory.

Data

RP with Choice Functions / Correspondences

Revealed preferences with choice functions The chosen element is *one* of the maximal alternatives:

$$\forall S \in 2^X, c^f(S) \in M(S)$$

Revealed preferences with choice correspondences The chosen set is the *set* of maximal alternatives:

$$\forall S \in 2^X, c^c(S) = M(S)$$

Setting

Increasing Chosen Sets and Identification ICS

Table 15: Proportion of subjects following increasing chosen sets for different gain pairs

	(no, low)	(no, high)	one or the other	identified
Sentence	82%	83%	88%	27%
Video	55%	60%	68%	24%
Training	59%	57%	73%	32%
All	60%	63%	**72%**	**26%**

References I

Agranov, Marina, and Pietro Ortoleva. 2017. “Stochastic Choice and Preferences for Randomization.” *Journal of Political Economy* 125 (1): 40–68. doi:10.1086/689774.

Aleskerov, Fuad, Denis Bouyssou, and Bernard Monjardet. 2007. *Utility maximization, choice and preference*. doi:10.1007/978-3-540-34183-3.

Arrow, Kenneth J. 1959. “Rational Choice Functions and Orderings.” *Economica* 26 (102): 121. doi:10.2307/2550390.

Aumann. 1962. “Utility Theory Without the Completeness Axiom.” *Econometrica* 30 (3). [Wiley, Econometric Society]: 445–62.

References II

http://www.jstor.org/stable/1909888.

Bouacida, Elias, and Daniel Martin. 2017. "Predictive Power in Behavioral Welfare Economics." https://halshs.archives-ouvertes.fr/halshs-01489252.

Choi, Syngjoo, Raymond Fisman, Douglas Gale, and Shachar Kariv. 2007. "Consistency and Heterogeneity of Individual Behavior Under Uncertainty." *American Economic Review* 97 (5): 1921–38. doi:10.1257/aer.97.5.1921.

Choi, Syngjoo, Shachar Kariv, Wieland Müller, and Dan Silverman. 2014. "Who Is (More) Rational?" *American Economic Review* 104 (6): 1518–50. doi:10.1257/aer.104.6.1518.

Costa-Gomes, Miguel, Carlos Cueva, Georgios Gerasimou, and Matus Tejiscak. 2016. "Choice, Deferral and Consistency."

Danan, Eric, and Anthony Ziegelmeyer. 2006. "Are Preferences Complete?: An Experimental Measurement of Indecisiveness Under Risk." Max-Planck-Inst. for Research into Economic Systems, Strategic

References III

Interaction Group. https://78462f86-a-4154f0ba-s-sites.googlegroups.com/a/ericdanan.net/www/research/preferences.pdf.

Danan, Eric, Thibault Gajdos, and Jean-Marc Tallon. 2013. "Aggregating Sets of von Neumann–Morgenstern Utilities." *Journal of Economic Theory* 148 (2): 663–88. doi:https://doi.org/10.1016/j.jet.2012.12.018.

Danan, Eric, Thibault Gajdos, Brian Hill, and Jean-Marc Tallon. 2016. "Robust Social Decisions." *American Economic Review* 106 (9): 2407–25. doi:10.1257/aer.20150678.

Dohmen, Thomas, Armin Falk, David Huffman, Uwe Sunde, Jürgen Schupp, and Gert G. Wagner. 2011. "Individual risk attitudes: Measurement, determinants, and behavioral consequences." *Journal of the European Economic Association* 9 (3): 522–50.

References IV

doi:10.1111/j.1542-4774.2011.01015.x.

Eliaz, Kfir, and Efe A. Ok. 2006. "Indifference or indecisiveness? Choice-theoretic foundations of incomplete preferences." *Games and Economic Behavior* 56 (1): 61–86. doi:10.1016/j.geb.2005.06.007.

Fishburn, Peter C. 1970. "Intransitive Indifference in Preference Theory: A Survey." *Operations Research* 18 (2): 207–28. doi:10.1287/opre.18.2.207.

Frick, Mira. 2016. "Monotone threshold representations." *Theoretical Economics* 11 (3): 757–72. doi:10.3982/TE1547.

Luce, R. Duncan. 1956. "Semiorders and a Theory of Utility Discrimination." *Econometrica* 24 (2): 178–91. doi:10.2307/1905751.

Nosratabadi, Hassan. 2017. "WARP Decompositions" 1248: 1–7.

Qiu, Jianying, and Qiyan Ong. 2017. "Indifference or Indecisiveness: a strict Discrimination," no. 8225.

References V

doi:10.1227/01.NEU.0000349921.14519.2A.

Samuelson, Paul A. 1938. "A Note on the Pure Theory of Consumer's Behaviour." *Economica* 5 (17): 61. doi:10.2307/2548836.

Sen, Amartya Kumar. 1971. "Choice Functions and Revealed Preference." *The Review of Economic Studies* 38 (3): 307–17. doi:10.2307/2296384.